Normal coverings of permutation groups and partitions

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Let G be the symmetric group or the alternating group of degree $n \in \mathbb{N}$. The *normal covering number* of G, denoted by $\gamma(G)$, is the minimum number of proper subgroups $H_i,\ i = 1, \dots, k$ of G such that each element in G lies in some conjugate of one of the H_i. Using some deep number theoretic results on the theory of *partitions*, it was proved in [1] that there exists a positive constant c such that $\gamma(G) \geq cn$. This method has allowed for the first time to get a lower bound for $\gamma(G)$ linear in n. However, number theory on its own provides estimates on c that are unrealistically small when compared to computational data.

Recently, in [2] and [3], finite primitive groups containing a permutation splitting in at most 4 cycles have been classified. This classification, together with some further results on 3-partitions with *clusters*, can be applied to derive a more realistic linear lower bound for $\gamma(G)$, when G is the symmetric group of even degree or the alternating group of odd degree ([4]).

In principle, a similar technique could be used on 4-partitions to analyze the symmetric group of odd degree and the alternating group of even degree. The obstructions for that idea come, at the moment, from difficulties of pure number theoretic nature in dealing with the intersection of 4-partitions with clusters, and from the fact that no closed formula for 4-partitions with globally coprime terms is available.

That facts confirm the fine interlacement between partition theory and normal coverings of permutation groups.

[1] D. Bubboloni, C. E. Praeger, P. Spiga, *Normal coverings and pairwise generation of finite alternating and symmetric groups*, J. Algebra **390** (2013), 199–215.

[2] S. Guest, C. E. Praeger, P. Spiga, *Finite primitive permutation groups containing a permutation having at most four cycles*, J. Algebra **454** (2016), 233–251.

[3] S. Guest, J. Morris, C. E. Praeger, P. Spiga, *Affine transformations of finite vector spaces with large orders or few cycles*, J. Pure and Applied Algebra **219** (2015), 308–330.

[4] D. Bubboloni, C. E. Praeger, P. Spiga, *Linear bounds for the normal covering number of the symmetric and alternating groups*, preprint (2018).